QUASI PERIODIC ORBITS IN THE VICINITY OF THE SUN-EARTH L_2 POINT AND THEIR IMPLEMENTATION IN “SPECTR-RG” & “MILLIMETRON” MISSIONS

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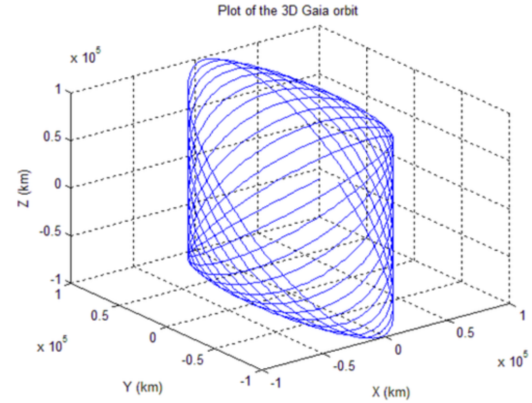
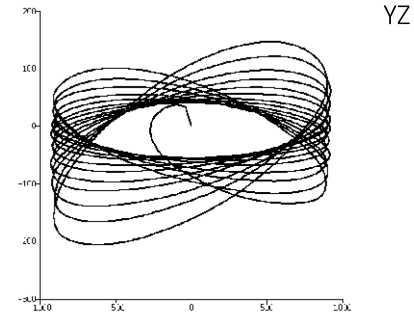
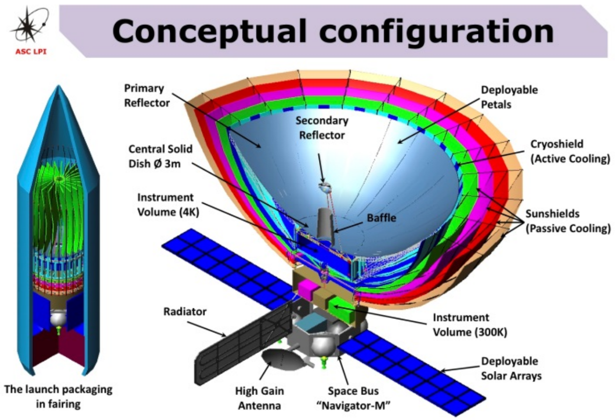
APPLICATION OF QUASI PERIODIC ORBITS NEAR LIBRATION POINTS

ESA space telescope **“Gaia”**
launched on 19.12.13,
direct transfer to a **Lissajous orbit** in the vicinity of the Sun-Earth L_2 point

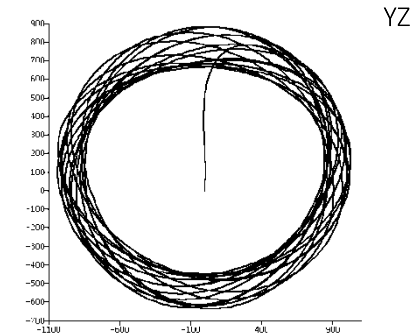


Roscosmos space telescope **“Spectr-RG”**
launch: scheduled at the end of 2016
direct transfer to a **Lissajous orbit** in the vicinity of the Sun-Earth L_2 point
Scientific mission: X-rays and Gamma range high precision sky survey, black holes, neutron stars, supernova explosions and galaxy cores study.

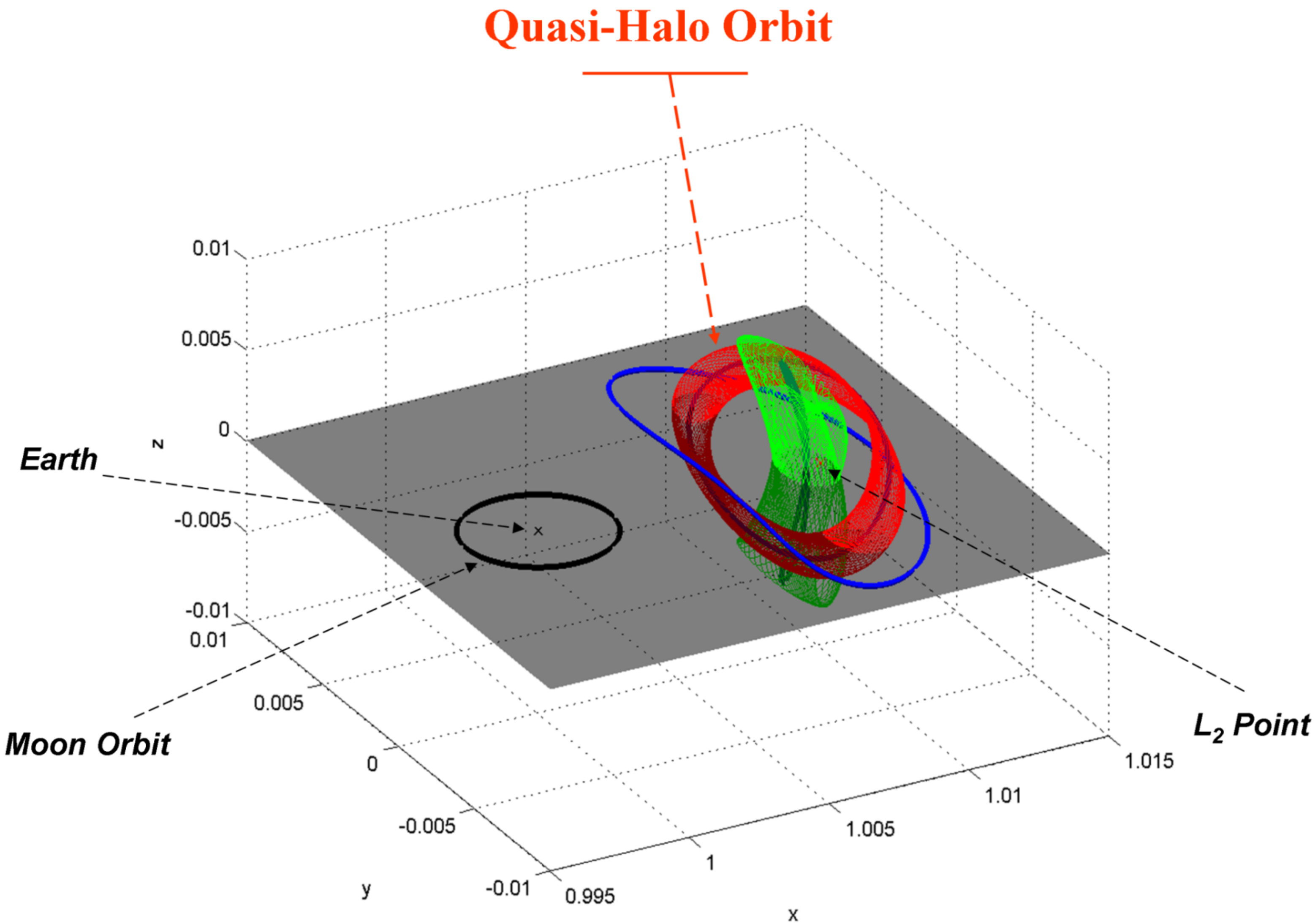




Roscosmos space telescope **“Millimetron”**
launch: scheduled in 2019 but may be pos
direct transfer to a **big radius quasi-halo orbit** in the vicinity of the Sun-Earth L_2 point
Scientific mission: space observation in millimeter, sub-millimeter and infrared ranges. The 12m space telescope will operate at cryogenic temperatures near 4K providing unique sensibility.



PERIODIC AND QUASI-PERIODIC MOTIONS AROUND L2 POINT



THREE BODY PROBLEM APPROXIMATION

1. Solution of the system of the linearized equations, describing circular restricted TBP

$$\xi_1 = A(t)\cos(\omega_1 t + \varphi_1(t)) + C(t)e^{\lambda t},$$

$$\xi_2 = -k_2 A(t)\sin(\omega_1 t + \varphi_1(t)) + k_1 C(t)e^{\lambda t}$$

$$\xi_3 = B(t)\cos(\omega_2 t + \varphi_2(t)),$$

$$A_{\min} \le A(t) \le A_{\max}$$

$$B_{\min} \le B(t) \le B_{\max}$$

$$|C(t)| \le C_{\max}$$

2. Ricahrdson 3d order analytical approximation of periodic motion about the collinear points, obtained with the help of Lindstedt-Poincaree technique applied to Legendre polinomial expansion of the classical CRTBP equations of motion

$$x = a_{21}A_x^2 + a_{22}A_z^2 - A_x\cos\tau_1 + (a_{23}A_x^2 - a_{24}A_z^2)\cos 2\tau_1 + (a_{31}A_x^3 - a_{32}A_xA_z^2)\cos 3\tau_1$$

$$y = kA_x\sin\tau_1 + (b_{21}A_x^2 - b_{22}A_z^2)\sin 2\tau_1 + (b_{31}A_x^3 - b_{32}A_xA_z^2)\sin 3\tau_1$$

$$z = \delta_n A_z\cos\tau_1 + \delta_n d_{21}A_xA_z(\cos 2\tau_1 - 3) + \delta_n(d_{32}A_zA_x^2 - d_{31}A_z^3)\cos 3\tau_1$$

$$\delta_n = 2 - n, \quad n = 1,3 \qquad A_z \ge 0 \qquad A_x > 0 \qquad A_{x\min} \ge Q$$

QUASI PERIODIC SOLUTION IN RTBP
TRANSITION FROM CIRCULAR RTBP TO ELLIPTIC RTBP

CRTBP

- Quasi-periodic orbit approximation: **Richardson model**

↓

- State vector $\vec{X}(t)$, lying on the obtained quasi periodic solution

$$x = \rho\xi \qquad y = \rho\eta \qquad z = \rho\zeta$$

$$\rho = \frac{p}{(1 + e\cos f)}$$

$$\frac{dx}{df} = \frac{dx}{dt}\cdot\frac{dt}{df}$$

$$\frac{dt}{df} = \frac{p^{3/2}}{(1 + e\cos f)^2}$$

$$\xi' = \dot{x}\frac{p^{3/2}}{(1 + e\cos f)} + x(-e\sin f)$$

$$\eta' = \dot{y}\frac{p^{3/2}}{(1 + e\cos f)} + y(-e\sin f)$$

$$\zeta' = \dot{z}\frac{p^{3/2}}{(1 + e\cos f)} + z(-e\sin f)$$

ERTBP

- Libration points in the RTBP are homographic, which means they keep their relative position when transfer to the ERTBP is performed

↓

- Transfer from dimensionless true anomaly ***f***, describing evolution of the elliptic system, to the dimensionless time ***t*** of the TBP is performed

$$tg\left(\frac{E}{2}\right) = \frac{tg\left(\frac{f}{2}\right)}{\sqrt{\frac{1+e}{1-e}}}$$

↓

$$M = E - e\sin E$$

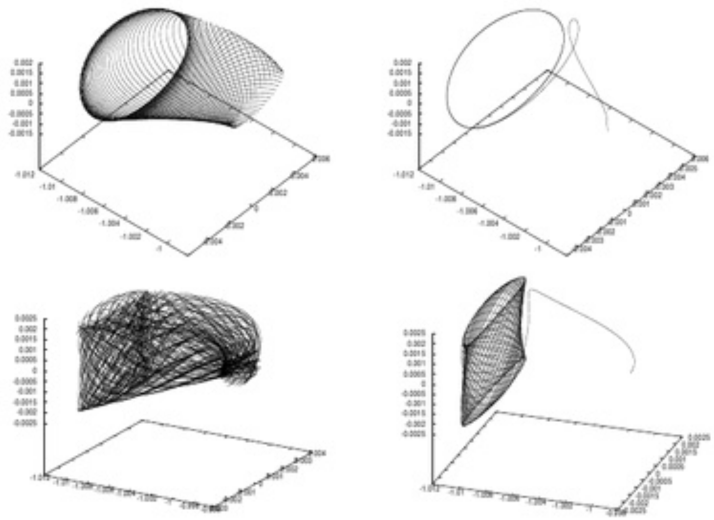
$$t_{dimensionless} = M$$

- A periodic halo orbit approximation is built with the help of **Richardson model**

↓

- TBP state vector $(x, y, z, \dot{x}, \dot{y}, \dot{z})$ is converted to Nehvill dimensionless variables $(\xi, \eta, \zeta, \dot{\xi}, \dot{\eta}, \dot{\zeta})$

TRANSFER TRAJECTORY DESIGN - THE ISOLINE METHOD



The transfer trajectory to the selected quasi-periodic orbit is searched within the invariant manifold of the restricted three body problem with help of the isoline method. This method provides connection between periodic orbit dots and geocentric transfer trajectory parameters – the isoloines of transfer trajectory pericentre height function depending on periodic orbit parameters are built. This provides one-impulse transfer from LEO to the quasi-periodic orbit.

$\vec{x}(x, y, z, \dot{x}, \dot{y}, \dot{z})$ $\quad \frac{17}{24} \cdot r_L \quad$ $\vec{\xi}(\xi, \eta, \zeta, \dot{\xi}, \dot{\eta}, \dot{\zeta})$ $\quad r_L$

Earth → L_2

LEO parameters: $r_\pi, r_\alpha, i, \Omega, \omega, \tau$

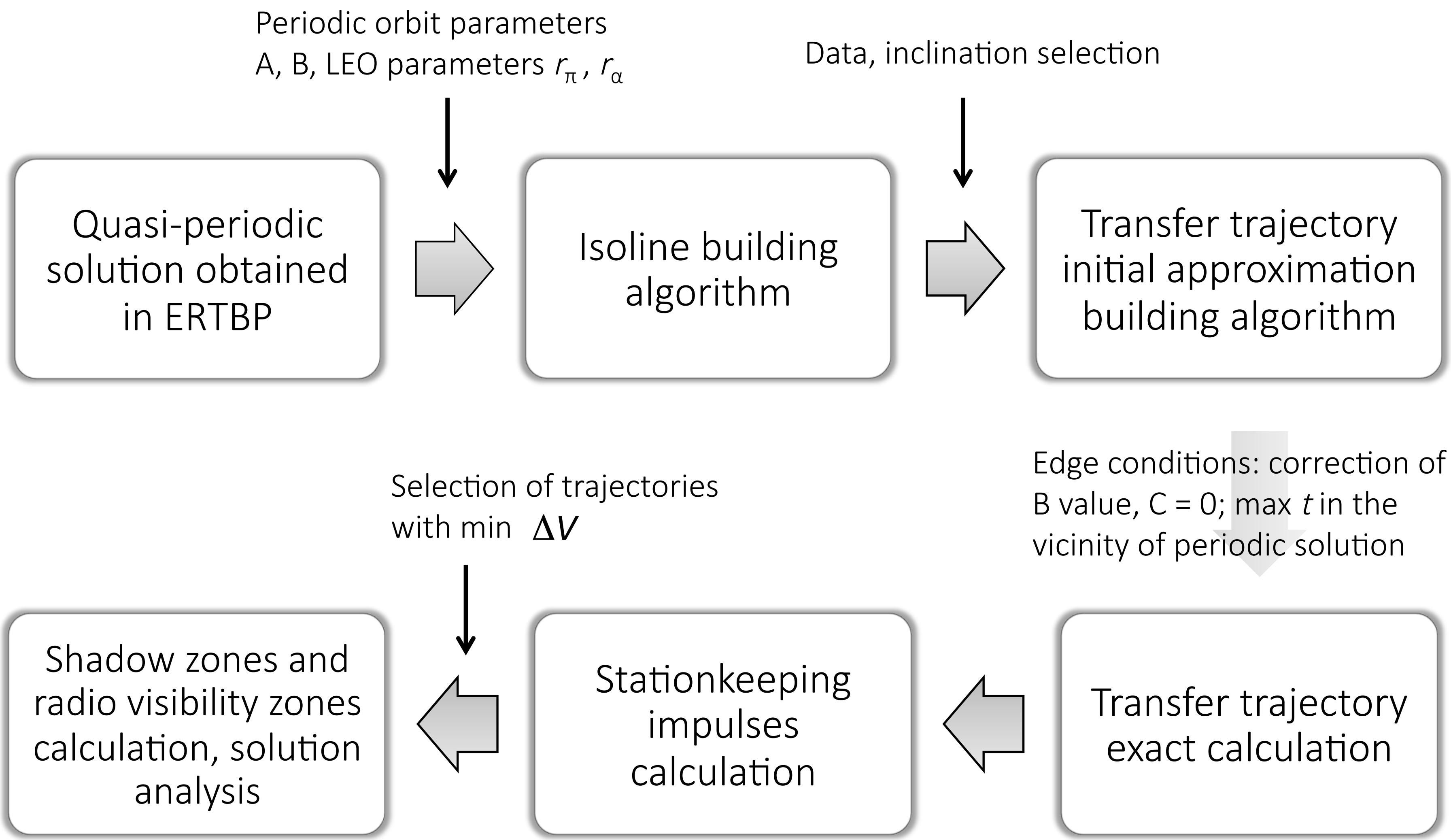
Periodic orbit parameters:

$A, B, C, D, \phi_1, \phi_2$

1-impulse flight trajectories are separated out with the condition: $r_\pi = r_\pi{}^*$

With the fixed *A, B* and *C* = 0 the isoline is built in the ϕ1, ϕ2 plane: $r_\pi(\phi_1, \phi_2) = r_\pi{}^*$

SPACECRAFT TRAJECTORY CALCULATION ALGORITHM STRUCTURE



STATIONKEEPING IMPULSES CALCULATION

The correction impulse vector is calculated according to the condition of the maximum time of the spacecraft staying in the L_2 point vicinity of the stated radius after the correction has been implemented. The maximum time is searched for with the help of the gradient method.

$$\Delta \vec{V}_i = \frac{1}{2^q} \frac{\Delta V \max}{\left|\nabla F_c\right|} \left(\nabla F_c\right)^T$$

ΔV_{max} - the biggest possible value of the impulse;

q - step decrease controlling coefficient.

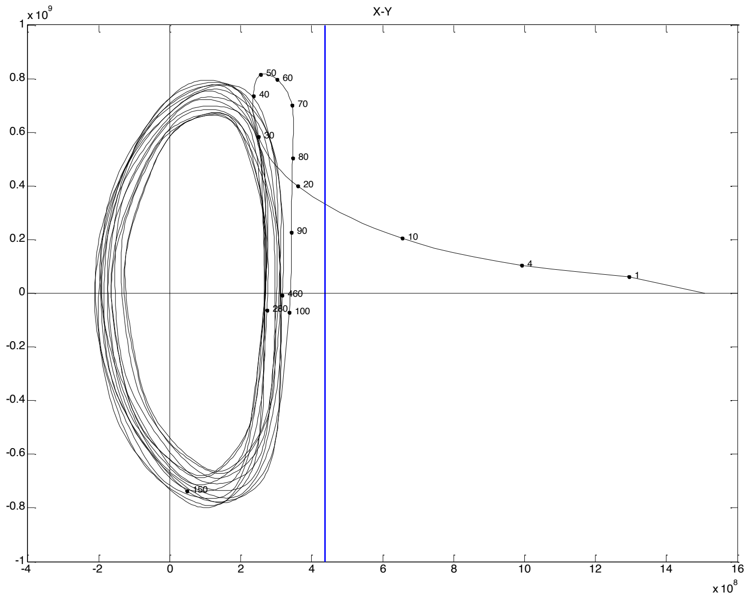
$$\mathrm{R}\left(\theta_{\mathrm{A}}, \theta_{\mathrm{B}}\right) = r_L \cdot \sqrt{\theta_{\mathrm{B}}^2 + \left(1 + k_2^2\right)\theta_{\mathrm{A}}^2}$$

$$F_C = \begin{cases} t_{\mathrm{outL2}} - t_{\mathrm{inL2}} \\ \dfrac{1}{T} \displaystyle\int_{t_1}^{t_1+T} \left(\left(B(t) - \theta_B r_L\right)^2 + C(t)^2\right) dt \end{cases}$$

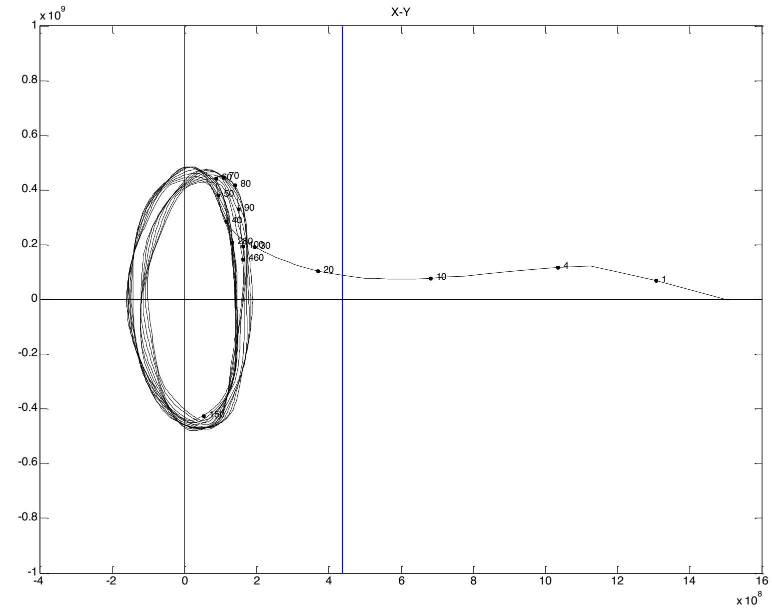
THE ISOLINE METHOD FOR THE MOON SWING BY TRANSFERS

Advantages: Moon swing by maneuver allows to obtain more compact orbit

Disadvantages: Trajectories become time-dependent and mission robustness decreases as the maneuver execution errors may cost a lot of ΔV to correct



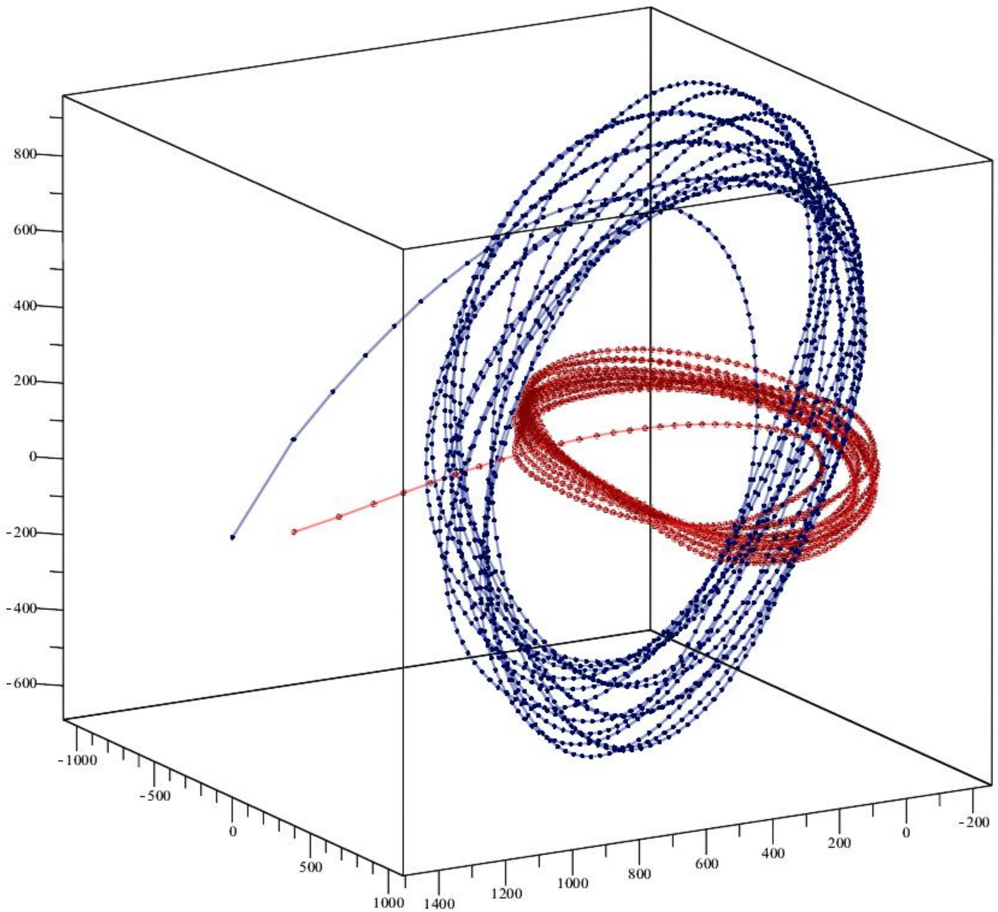
Quasi-periodic orbit obtained without the Moon swing by maneuver
Ay = 0.8 million km

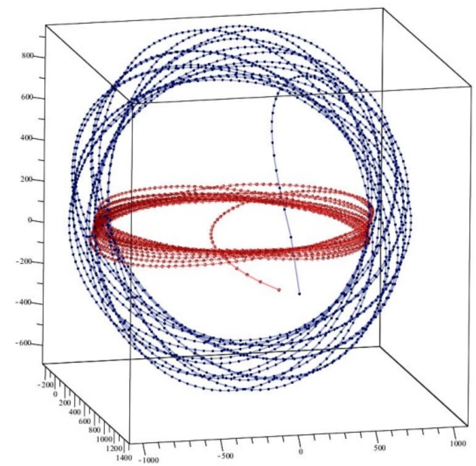


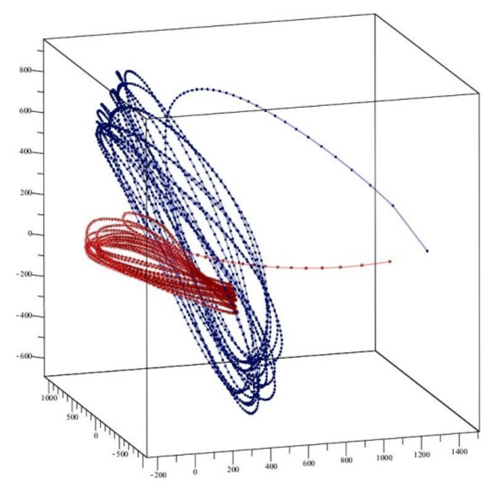
Quasi-periodic orbit obtained with the help of Moon swing by maneuver
Ay = 0.5 million km

The XY plane view of the rotating reference frame, mln km.

QUASI PERIODIC ORBITS
FOR "SPECTR-RG" & "MILLIMETRON" MISSIONS



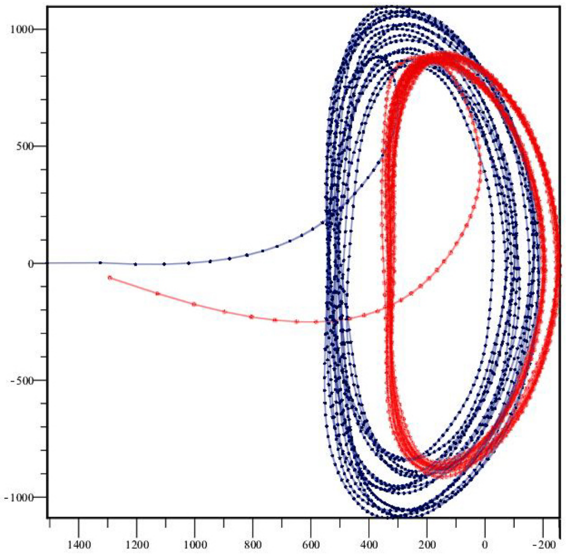




Total ΔV costs are less than 15 m/s for 7 years period.

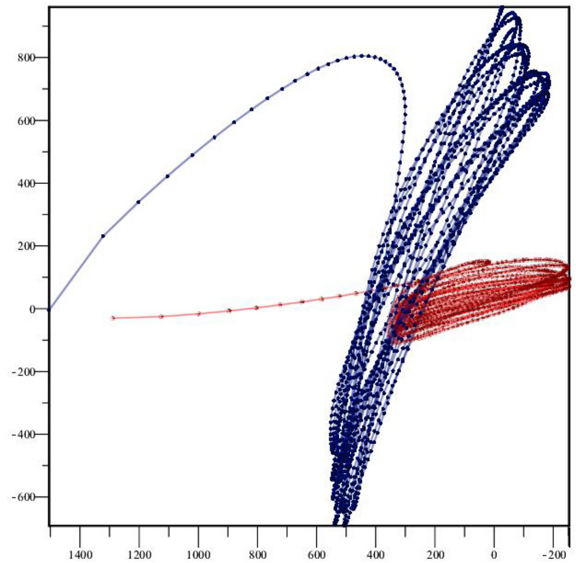
QUASI PERIODIC ORBITS
FOR “SPECTR-RG” & “MILLIMETRON” MISSIONS
XY, XZ, YZ PROJECTIONS ON THE ROTATING REFERENCE FRAME

1000



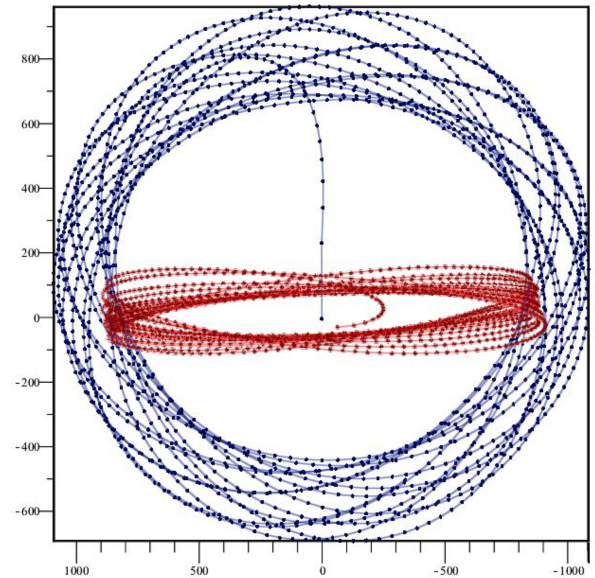
-1000 -200

1000



-600 -200

1000



-600 -1000

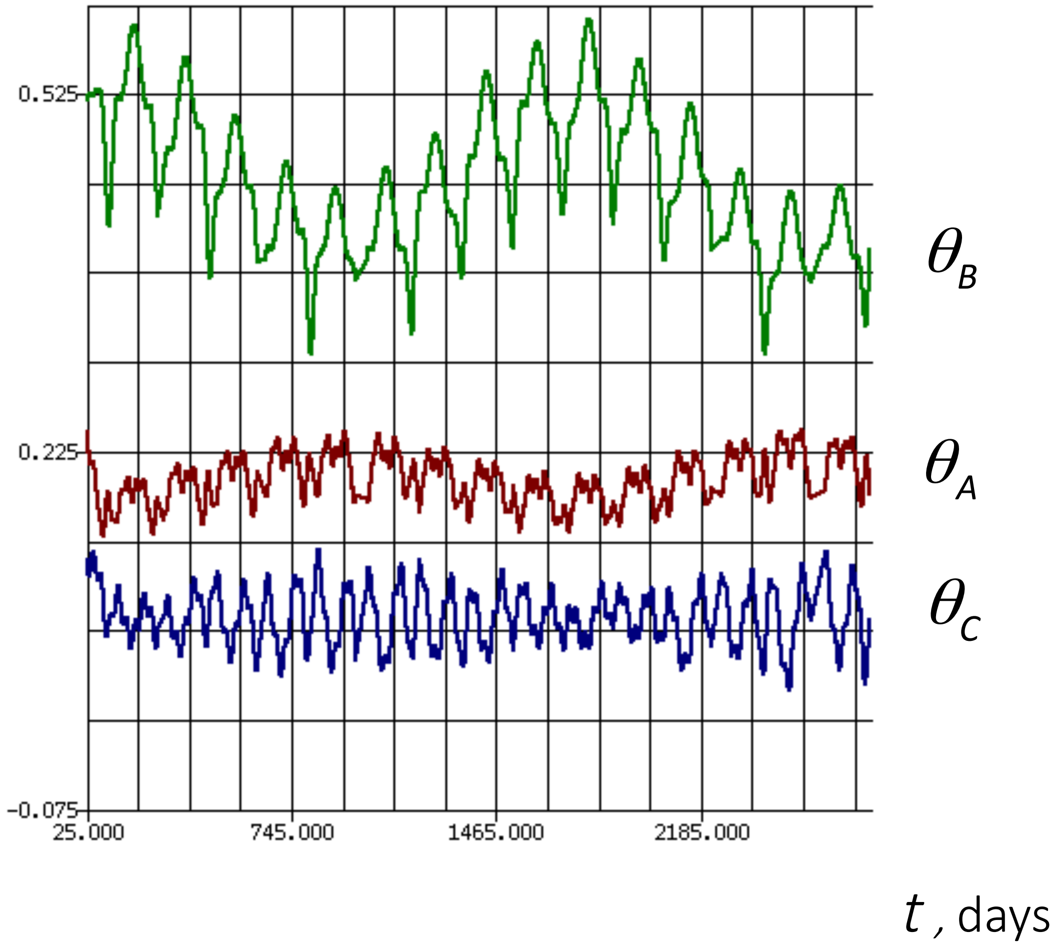
Dimension: thousands of km

EVOLUTION OF DIMENSIONLESS PARAMETERS DESCRIBING THE QUASI-PERIODIC ORBIT GEOMETRY

$$\theta_A = \frac{A}{R_L}$$

$$\theta_B = \frac{B}{R_L}$$

$$\theta_C = \frac{C}{R_L}$$

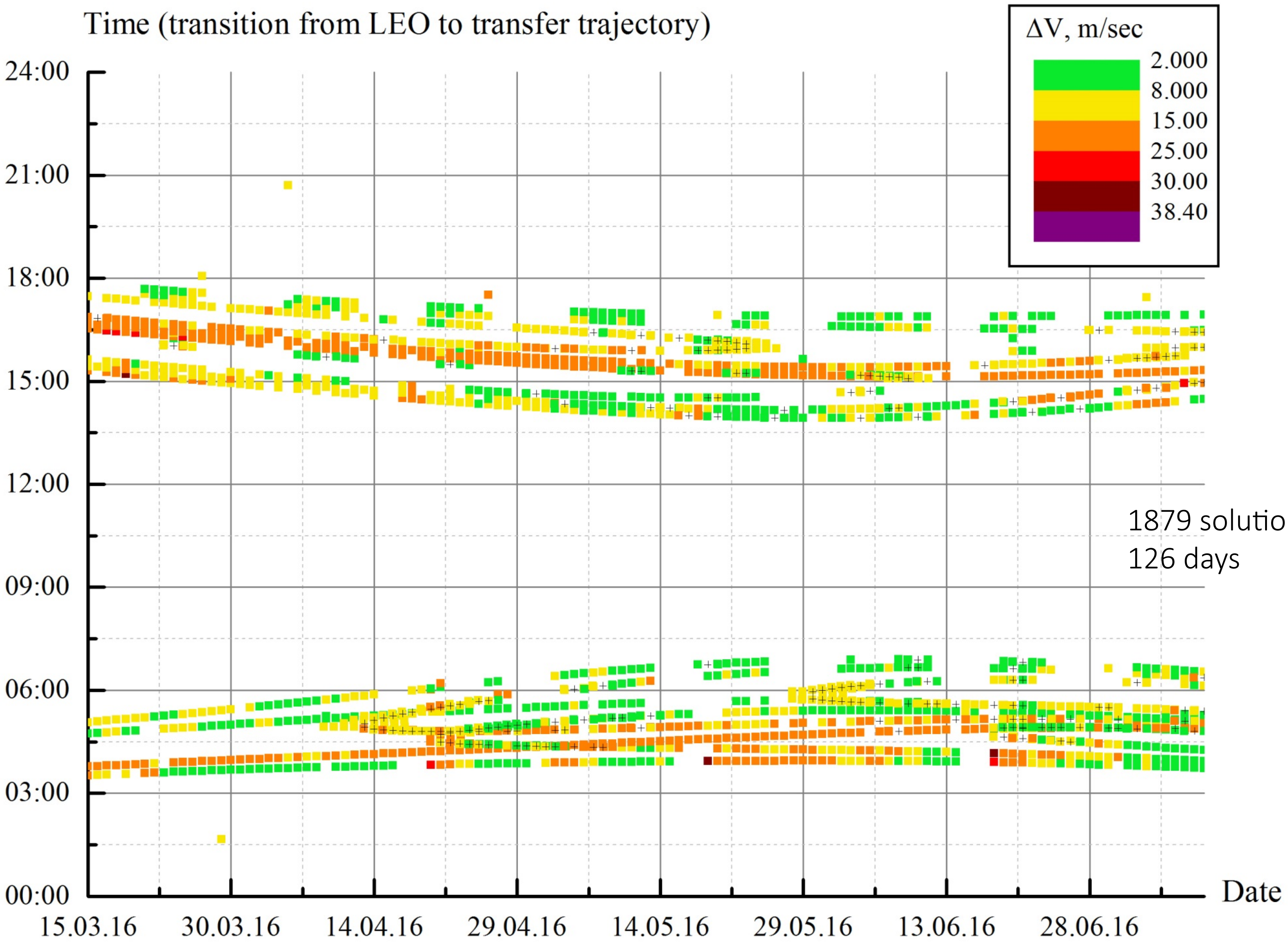


MISSIONS' CONSTRAINTS

Mission	Spectr-RG		Millimetron	
Maneuver	Av, m/sec	Max, m/sec	Av, m/sec	Max, m/sec
1st MCC	22.2	43.8	22.2	43.8
2nd MCC	2.4	10.1	3.3	13.5
3d MCC	0.3	1.3	0.4	1.6
4th MCC	1.2	2.8	1.4	3.0
Stationkeeping ΔV costs	35.0	124.0	43.0	173.0
Total ΔV costs	61.1	182.0	70.3	234.9

Quasi-periodic orbit parameters	Requirements/ Constraint Driver(s)	Spectr-RG	Millimetron
Orbit geometry: Y amplitude	Sp-RG: Communications Millimetron: Science	Maximum 900.000 km	Minimum 900.000 km
Orbit geometry: Z amplitude	Sp-RG: Communications Millimetron: Science	Maximum 600.000 km	Minimum 900.000 km
Maximum SCE finite-burn duration	Propulsion	1800.0 sec	2000.0 sec
Minimum precision of SCE finite-burn duration	Propulsion	0.1 sec	0.2 sec
Estimated MCC average ΔV	Mass & Propulsion	55 m/s	28 m/s
Stationkeeping available ΔV	Mass & Propulsion	228 m/s	287 m/s
Mission lifetime goal	Science	7.5 years	7.5 years
Lunar / Earth Eclipse	Power and Thermal	None allowed	None allowed
Radio visibility	Communications, navigation & control	Must be provided every day for Northern hemisphere ground stations	Southern hemisphere ground stations should be used

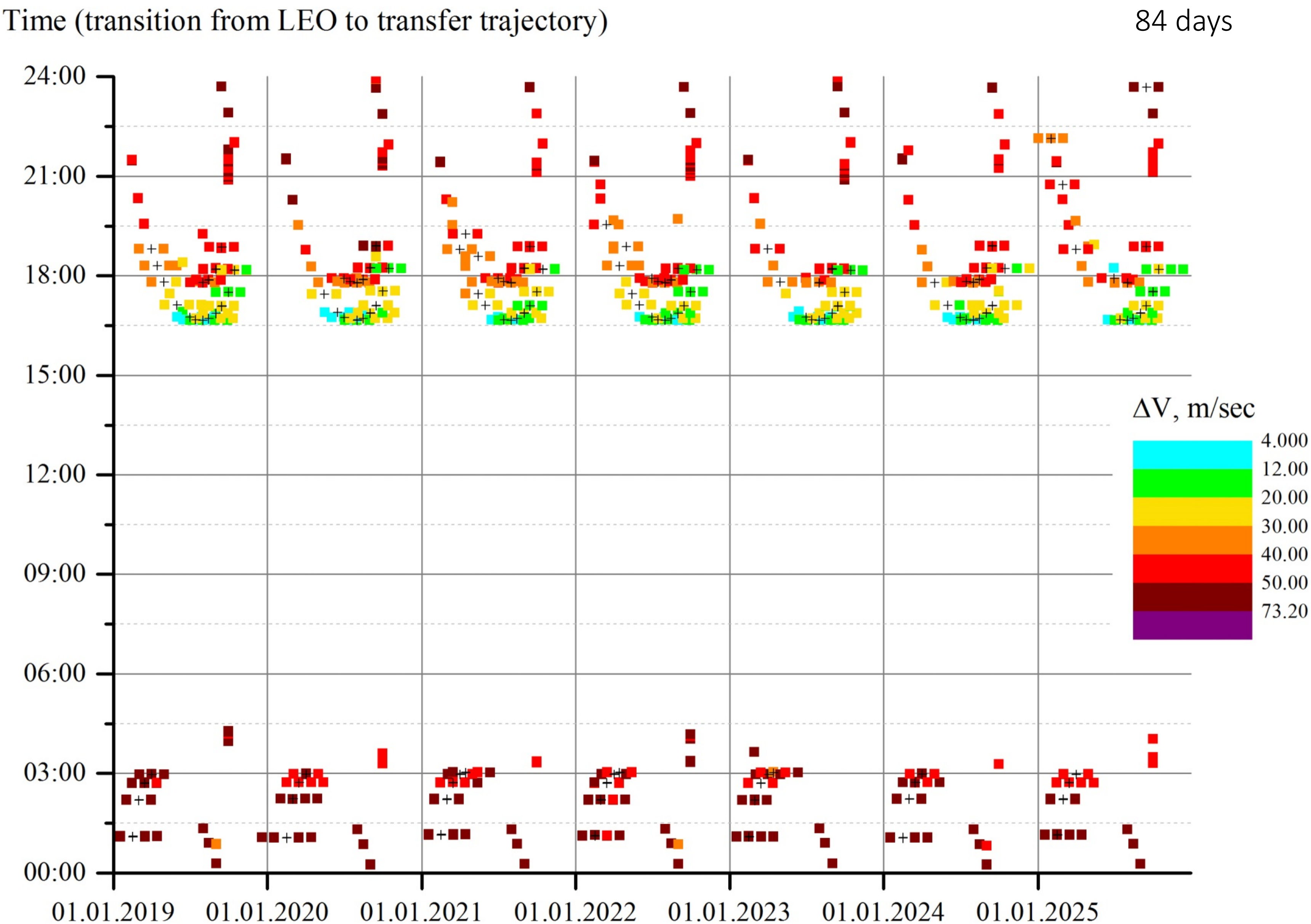
SPECTR-RG SOLUTIONS MAP



1879 solutions
126 days

MILLIMETRON SOLUTIONS MAP

904 solutions
84 days



RESEARCH RESULTS

- A new method of quasi periodic orbits construction, generalizing Lindstedt-Poincaree-Richardson technique for the ERTBP case has been developed and programmed.
- M.L. Lidov's isoline building method providing one-impulse transfers from LEO to a quasi-periodic orbit in the vicinity of a collinear libration point has been extend on gravity assist trajectory class.
- An algorithm calculating stationkeeping impulses for the quasi periodic orbit maintainance has been developed and programmed. It provides stationkeeping strategies for spacecraft lifetime over 7 years, total ΔV costs are within 15 m/sec.
- Nominal trajectories for Spectr-RG and Millimetron missions have been obtained by performing the calculation described above in the full Solar system ballistic model. All the restrictions such as Earth and Moon shadow avoidance conditions and constant radio visibility from the Northern hemisphere have been met.
- Engine error modeling has been performed, the most pessimistic scenario results satisfy both mission's constraints.