Determination and Prediction of Orbital Parameters of the Radioastron Mission

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May 9, 2014
Radioastron mission

### Orbit

<table>
<thead>
<tr>
<th>Launch date</th>
<th>18 July 2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perigee altitude</td>
<td>1000 – 67 000 km</td>
</tr>
<tr>
<td>Apogee distance</td>
<td>up to 370 000 km</td>
</tr>
<tr>
<td>Period</td>
<td>8–9 days</td>
</tr>
</tbody>
</table>

### Mission

<table>
<thead>
<tr>
<th>Purpose</th>
<th>VLBI observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bands</td>
<td>P, L, C, K</td>
</tr>
<tr>
<td>Data channel</td>
<td>2x72 Mbps</td>
</tr>
</tbody>
</table>

Orbit knowledge is required for the interferometric data correlation.

### Main perturbations

<table>
<thead>
<tr>
<th>Nature</th>
<th>Maximum, m/s²</th>
<th>Average, m/s²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spherical harmonics</td>
<td>$3.8 \cdot 10^{-3}$</td>
<td>$3.3 \cdot 10^{-6}$</td>
</tr>
<tr>
<td>Third bodies</td>
<td>$2.3 \cdot 10^{-4}$</td>
<td>$4.1 \cdot 10^{-5}$</td>
</tr>
<tr>
<td>Direct solar radiation</td>
<td>$1.9 \cdot 10^{-7}$</td>
<td>$1.5 \cdot 10^{-7}$</td>
</tr>
<tr>
<td>Unloadings</td>
<td>$5.8 \cdot 10^{-8}$</td>
<td>$5.8 \cdot 10^{-8}$</td>
</tr>
<tr>
<td>Tides</td>
<td>$6.6 \cdot 10^{-8}$</td>
<td>$2.3 \cdot 10^{-11}$</td>
</tr>
<tr>
<td>Earth radiation</td>
<td>$2.1 \cdot 10^{-8}$</td>
<td>$1.1 \cdot 10^{-10}$</td>
</tr>
</tbody>
</table>

The satellite is not equipped with accelerometers.
Unloadings of reaction wheels

An unloading consists of dozens of firings, \( j \)-th firing of the \( i \)-th unloading provides \( \Delta v_{ij} = \frac{\Delta m_{ij} I(\tau_{ij})}{M} e_{ij} \) at \( t_{ij} \).

All firings of the unloading are summed up into one impulse \( \Delta v_i \) applied at weighted time \( t_i \).

Weighted time and the estimate of the impulse are as follows

\[
\{ \Delta v_{ij}, t_{ij} \} \quad \Rightarrow \quad \Delta v_i, t_i
\]

\[
t_i = \frac{\sum_j v_{ij} t_{ij}}{\sum_j v_{ij}} \quad v_i^0 = \sum_j v_{ij} (\Delta m_{ij}, \tau_{ij}, e_{ij}).
\]

The perturbation due to unloadings on the interval of interest is described with the set of impulses \( \{ \Delta v_i, t_i \} \).
Direct solar radiation pressure

Decomposition of the solar radiation impacting a flat surface

\[ \mathbf{F} = (1 - \alpha) \mathbf{F}_a + \alpha \mu \mathbf{F}_s + \alpha (1 - \mu) \mathbf{F}_d \]

\( \alpha \in [0, 1] \) — reflectivity,
\( \mu \in [0, 1] \) — specularity.

allows to represent net SRP force and torque as functions of parameters \( \alpha_i \) and \( \mu_i \)

\[ \mathbf{F}_{SRP} = \sum_{i=1}^{N} \eta_i \mathbf{F}(A_i, s, n_i, \alpha_i, \mu_i), \]

\[ \mathbf{M}_{SRP} = \sum_{i=1}^{N} \eta_i \mathbf{r}_i \times \mathbf{F}(A_i, s, n_i, \alpha_i, \mu_i), \]
The satellite structure

<table>
<thead>
<tr>
<th>Element</th>
<th>Surface</th>
<th>Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>space radio telescope</td>
<td>reflecting (MLI)</td>
<td>$\alpha_1, \mu_1$</td>
</tr>
<tr>
<td>spacecraft bus</td>
<td>reflecting (MLI)</td>
<td>$\alpha_1, \mu_1$</td>
</tr>
<tr>
<td>solar panels</td>
<td>absorbing</td>
<td>$\alpha_2 (\mu_2 = 1)$</td>
</tr>
</tbody>
</table>

specularity coefficient of solar panels is fixed to avoid strong correlation with $\alpha_2$
Propagation

passing through an unloading:

\[(t_{i-0}, \mathbf{r}(t_{i-0}), \mathbf{v}(t_{i-0}), \ldots) \rightarrow (t_{i+0}, \mathbf{r}(t_{i+0}), \mathbf{v}(t_{i-0}) + \Delta \mathbf{v}_i, \ldots),
\]
\[(t_{i-0}, \mathbf{r}(t_{i-0}), \mathbf{v}(t_{i+0}) - \Delta \mathbf{v}_i, \ldots) \leftarrow (t_{i+0}, \mathbf{r}(t_{i+0}), \mathbf{v}(t_{i+0}), \ldots).\]

Gravity field \hspace{1cm} EGM96
Third bodies \hspace{1cm} DE-405
Tides \hspace{1cm} IERS 2003 convention
Direct SRP \hspace{1cm} parameterized with \(\alpha_1, \mu_1\) and \(\alpha_2\)
Earth radiation \hspace{1cm} 18x9 constant coeff.

Motion of the center of mass is determined by:

\[\mathbf{X}_0(t_0), \alpha_1, \mu_1, \alpha_2, \Delta \mathbf{v}_1, \ldots, \Delta \mathbf{v}_n\]
Observations

Radio

- Two-way range, two-way Doppler
- One-way Doppler

<table>
<thead>
<tr>
<th>System</th>
<th>Band</th>
<th>( D )</th>
<th>( \dot{D} )</th>
<th>( \dot{D}_{1w} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ussuriysk RT-70, &quot;Klen-D&quot;</td>
<td>C</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Ussuriysk RT-70, &quot;Phobos&quot;</td>
<td>X</td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Bear Lakes RT-64, &quot;Cobalt-M&quot;</td>
<td>C</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Bear Lakes RT-64, &quot;Cortex&quot;</td>
<td>X</td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Puschino RT-22</td>
<td>X, Ku</td>
<td></td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Green Bank, 140ft</td>
<td>X, Ku</td>
<td></td>
<td></td>
<td>✓</td>
</tr>
</tbody>
</table>

Optical

- CCD RA/Dec: ISON, ASC

Telemetry

- Observed impulses of unloadings \( \Delta v_i^0 \).
- Observed torque \( \mathbf{M} \).
Perturbing torque observations

During constant attitude far from the Earth

\[
\sum_{i=1}^{8} a_i I_i (\Omega_i(t_2) - \Omega_i(t_1)) = M_{SRP}(\Lambda, \alpha_1, \mu_1, \alpha_2)(t_2 - t_1).
\]

Introduce the following difference of observed and computed torque

\[
\zeta = \sum_{i=1}^{8} \frac{a_i I_i [\Omega_i(t_2) - \Omega_i(t_1)]}{t_2 - t_1} - M_{SRP}(\Lambda, \alpha_1, \mu_1, \alpha_2).
\]
Orbit determination

Solve for the following parameters on the interval \([t_b, t_e]\)

\[ Q = \{X_0(t_0), \alpha_1, \mu_1, \alpha_2, \Delta \mathbf{v}_1, \ldots, \Delta \mathbf{v}_n\}. \]

using tracking data

\[ \Psi = \{D, \dot{D}, \dot{D}_{1w}, \alpha, \delta\} \]

and on-board observations

\[ \{\Omega(t), \Delta \mathbf{v}_1^0, \ldots, \Delta \mathbf{v}_n^0\} \]

to minimize the functional

\[
\Phi = (\Psi_o - \Psi_c)^T \mathbf{P} (\Psi_o - \Psi_c) + \sum_{j=1}^{N} \zeta_j^T \mathbf{P}_{sp} \zeta_j + \\
+ \sum_{i=1}^{n} (\Delta \mathbf{v}_i^0 - \Delta \mathbf{v}_i)^T \mathbf{P}_i (\Delta \mathbf{v}_i^0 - \Delta \mathbf{v}_i),
\]
Orbit determination

Two intervals have been considered:

- 20-Feb-2013 – 10-Apr-2013 (Int. 1)
- 10-Apr-2013 – 30-May-2013 (Int. 2)

Several models were used:

- Simple SRP, No unloadings.
- Simple SRP, unloadings fixed on their nominal values $\Delta v_i^0$
- SRP depends on three coefficients, unloadings fixed on their nominal values $\Delta v_i^0$
- SRP depends on three coefficients, unloadings are solved for.

Parameters obtained on the Int. 1 will be used for orbit prediction on the Int. 2.
Radioastron $D, \dot{D}, 20.02.13 - 10.04.13$

Range rate, O-C

Range, O-C

Puschino Bear Lakes Ussuriysk

Bear Lakes Ussuriysk
Radioastron ($\alpha$, $\delta$), 20.02.13 – 10.04.13

Right Ascension, O-C

Declination, O-C

Keypad:
- Red: Kitab
- Green: Mondy
- Orange: Uzhgorod
- Blue: Evpatoria
- Black: Krasnodar
- Purple: Blagoveshensk
Radioastron $\{\Delta v_i^0 - \Delta v_i \}$, 20.02.13 – 10.04.13
Radioastron $D, \dot{D}$, 10.04.13 – 30.05.13

Range rate, O-C

Range, O-C
Radioastron ($\alpha, \delta$), 10.04.13 – 30.05.13

Right Ascension, O-C

Declination, O-C

15.04.13 22.04.13 29.04.13 06.05.13 13.05.13 20.05.13 27.05.13

Kitab  H06, Mayhill  Uzhgorod  Blagoveshensk
Evpatoria  Krasnodar  Q62, Siding Springs AU
Radioastron $\{\Delta v_i^0 - v_i\}$, 10.04.13 – 30.05.13

Values of the impulses of unloadings, O-C

Directions of the impulses of unloadings, O-C
Orbit determination results

Dimensionless standard deviation: 20.02.13 – 10.04.13 (Int. 1) and 10.04.13 – 30.05.13 (Int. 2)

<table>
<thead>
<tr>
<th>№</th>
<th>SRP model</th>
<th>Unloadings</th>
<th>$\sigma_1$</th>
<th>$\sigma_2$</th>
<th>$\Delta r$, km</th>
<th>$\Delta v$, mm/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Simple, 1 coeff.</td>
<td>Not considered</td>
<td>12.43677</td>
<td>9.18588</td>
<td>71.71</td>
<td>288.1</td>
</tr>
<tr>
<td>2</td>
<td>Simple, 1 coeff.</td>
<td>Nominal</td>
<td>4.72914</td>
<td>6.78832</td>
<td>36.76</td>
<td>113.3</td>
</tr>
<tr>
<td>3</td>
<td>Proposed, 3 coeff.</td>
<td>Nominal</td>
<td>1.20896</td>
<td>0.63767</td>
<td>7.57</td>
<td>8.9</td>
</tr>
<tr>
<td>4</td>
<td>Proposed, 3 coeff.</td>
<td>Solved for</td>
<td>0.28198</td>
<td>0.24907</td>
<td>0.21</td>
<td>2.3</td>
</tr>
</tbody>
</table>

Estimated solar radiation pressure coefficients

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Int. 1</th>
<th>Int. 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>0.754</td>
<td>0.791</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>0.087</td>
<td>0.089</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.063</td>
<td>0.102</td>
</tr>
</tbody>
</table>
Orbit prediction

Necessary elements

- Attitude forecast (observation schedule + service attitude)

\[(\Lambda_1, t_1, t'_1), (\Lambda_2, t_2, t'_2), \ldots (\Lambda_n, t_n, t'_n).\]

With estimated SRP coefficients determines corresponding perturbation and accumulation of angular momentum by the reaction wheels

- Conversion of accumulated angular momentum to impulses of unloadings

\[K(t) \xrightarrow{\Delta v(t)} K(t + \delta t)\]

- Prediction of times of occurrence of unloadings.
Angular momentum to unloading

Angular momentum changing during an unloading can be described as follows:

\[ \sum_{i=1}^{8} a_i I_i \Omega_i(t_u) = \sum_{j=1}^{4} r_j \times e_j p_j, \]

- an unloading takes relatively short time,
- reaction wheels stop,
- the satellite is not rotating.

where \( p_j \geq 0 \) are the propellant momenta. The equation can be resolved with respect to \( \{p_j\} \) with additional condition:

\[ \sum_{j} p_j \rightarrow \text{min}. \]

An impulse of an unloading

\[ \Delta v^*(K) = - \frac{\sum_i p_i e_i}{M}, \quad \Delta v = \Delta v(\Delta v^*). \]
Prediction of the time of next unloading

An unloading should be conducted if accumulated angular momentum is too high \( K(t) \notin U \)

\[
U = \left\{ K = \sum_{i=1}^{N} a_i I_i \Omega_i : |\Omega_i| \leq \Omega_{max}, i = 1, N \right\}.
\]

Unloadings can be conducted on daily basis in the same time.
Prediction of unloadings on the Int. 2 (10-Apr-2013 – 30-May-2013)

The graph shows the measured angular momentum (black line) and the calculated angular momentum (blue line) over time. The yellow bars represent "forced" unloadings, and the red bars represent "regular" unloadings. The data is presented from 16.04.13 to 21.05.13.
Results of orbit prediction

Along-track error

- Red line: no unloadings
- Blue line: "regular" unloadings
- Green line: "forced" unloadings

\[ \Delta R_t, \text{km} \]

Days

Along-track error

- Black line: "regular" unloadings
- Orange line: "forced" unloadings
- Blue line: actual unloadings

\[ \Delta R_t, \text{km} \]
Summary

- Adjustable Radioastron SRP model was developed and tested.
- Parameters of the SRP model was estimated by using both motion of the center of mass and motion around the center of mass.
- Determined orbits are successfully used for correlation of the Radioastron observations.
- An unloading prediction approach, important for future Sun-Earth $L_2$ missions (Spectr-R, 'Millimetron) based on the same platform, was tested on the Radioastron data.
Thank you for your attention!